

# Signal Processing

2011

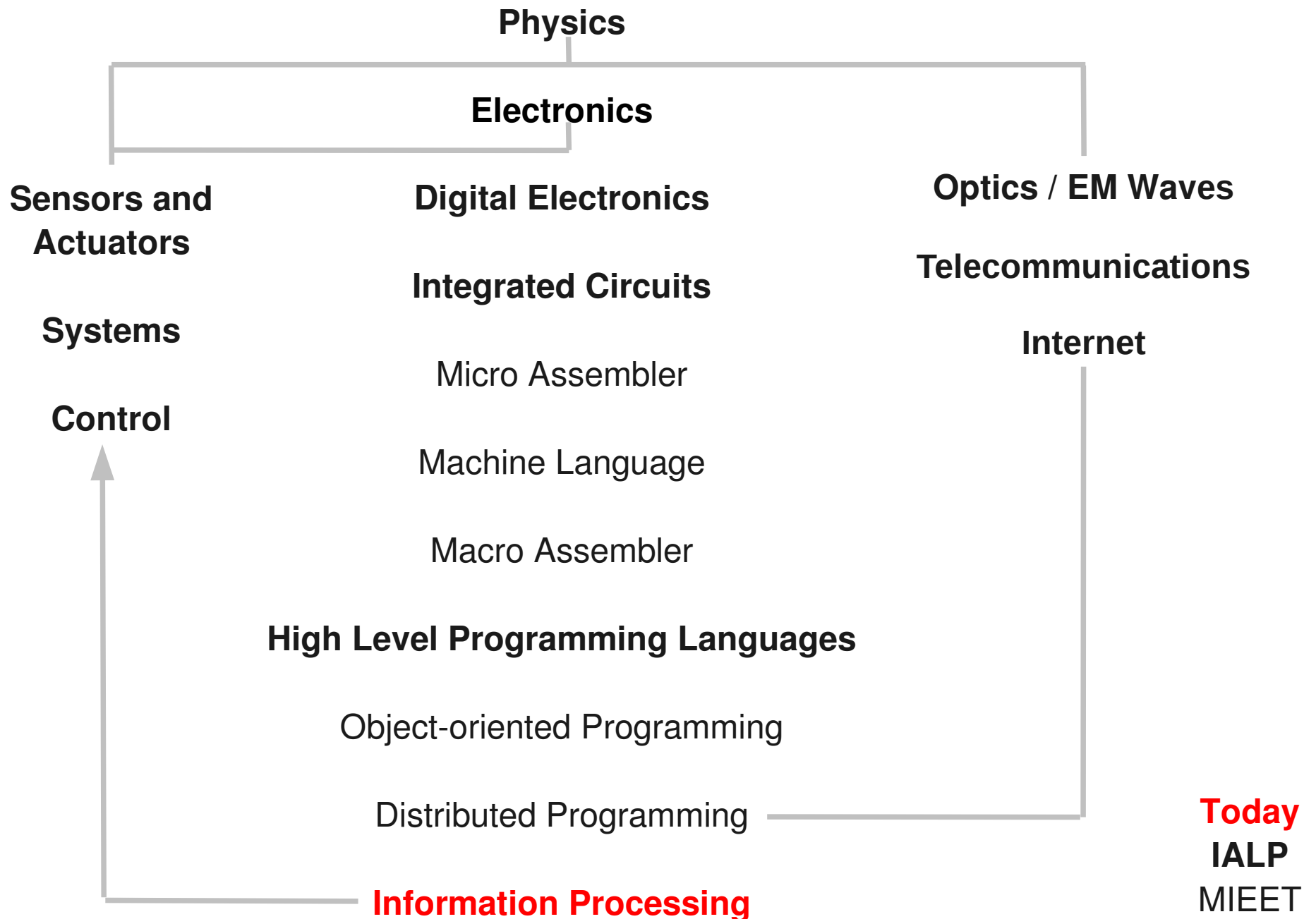
MIEET

1<sup>o</sup> ano



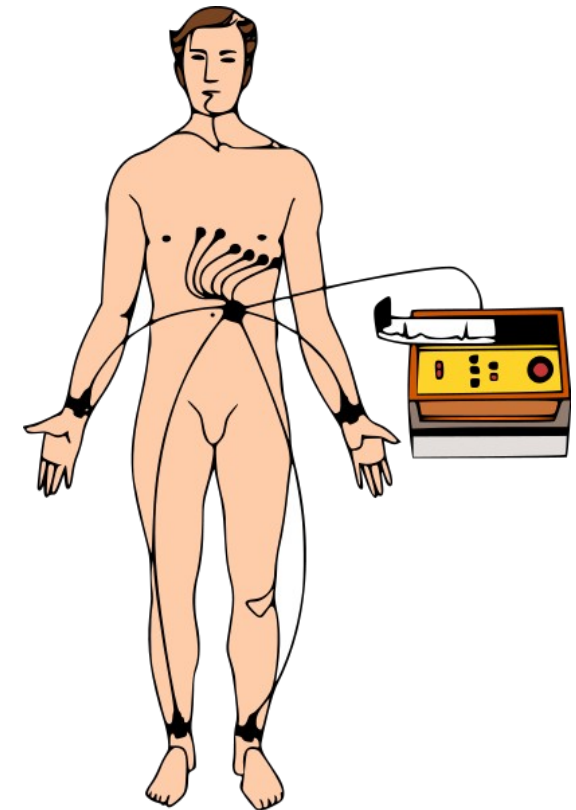
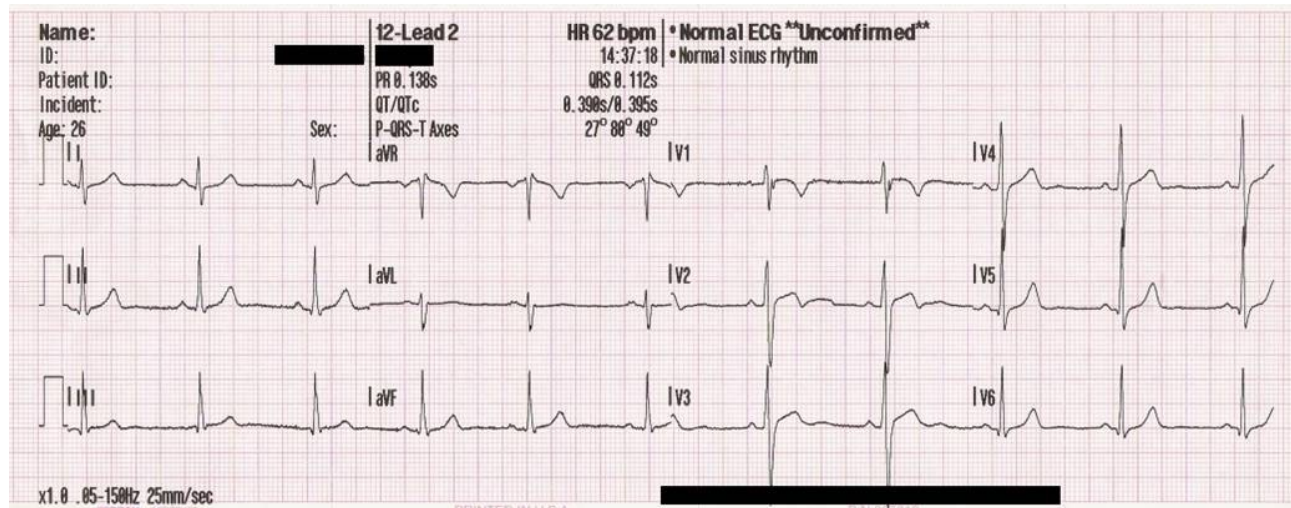
Peter Stallinga, UAlg 2011

# MIEET. The levels of knowledge



# Signal processing. Example: cardiogram

Electrocardiography:



Is this person healthy?

**The information is there somehow, but how to extract it?**

# Signal processing

Frequency analysis

Filtering

Noise

Fourier/Laplace Transform

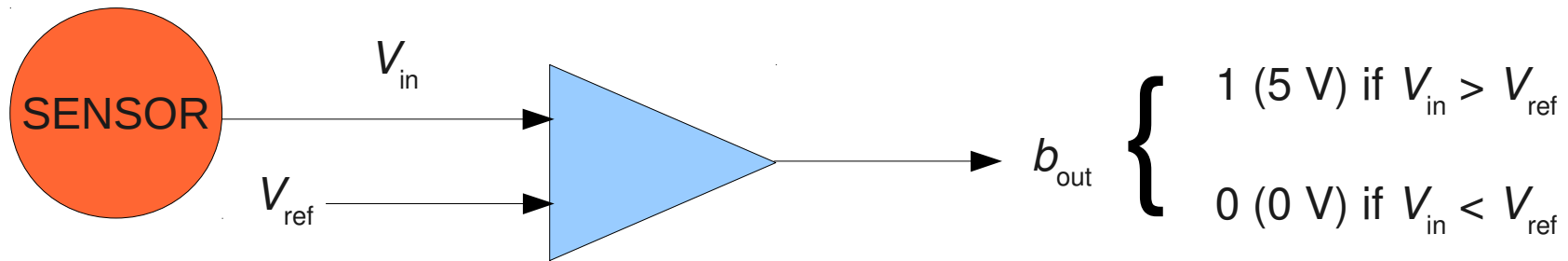
(Pattern recognition)

# ADC/DAC

ADC (Analog-Digital converter)

DAC (Digital-Analog Converter)

Translate signal from analog to digital domain and back

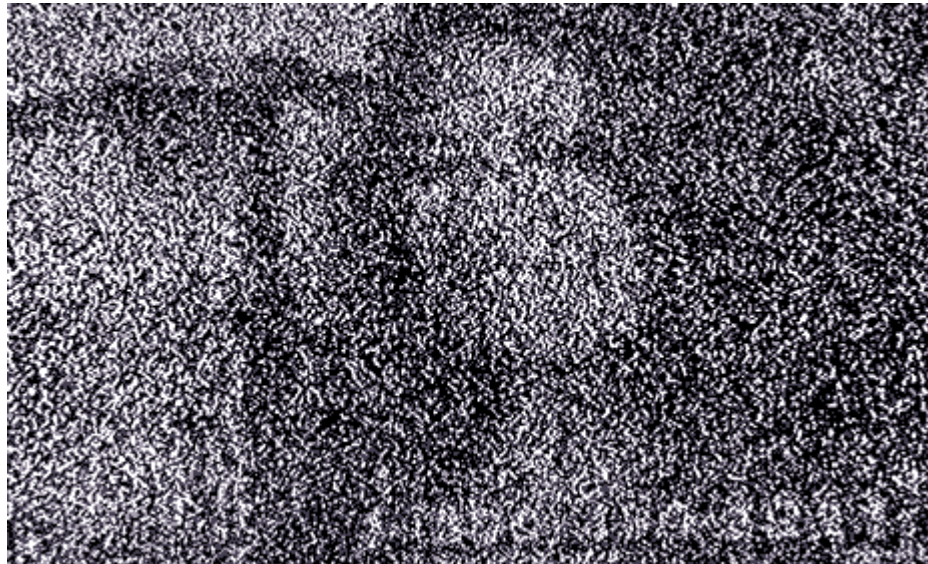


Simple 1-bit ADC:  
comparator

# Signal, Noise and S/N

**Signal** is that part of the incoming voltages (currents) that contain the **useful information**

**Noise** is that part of the incoming voltages (or currents) that contain non meaningful or **no information**

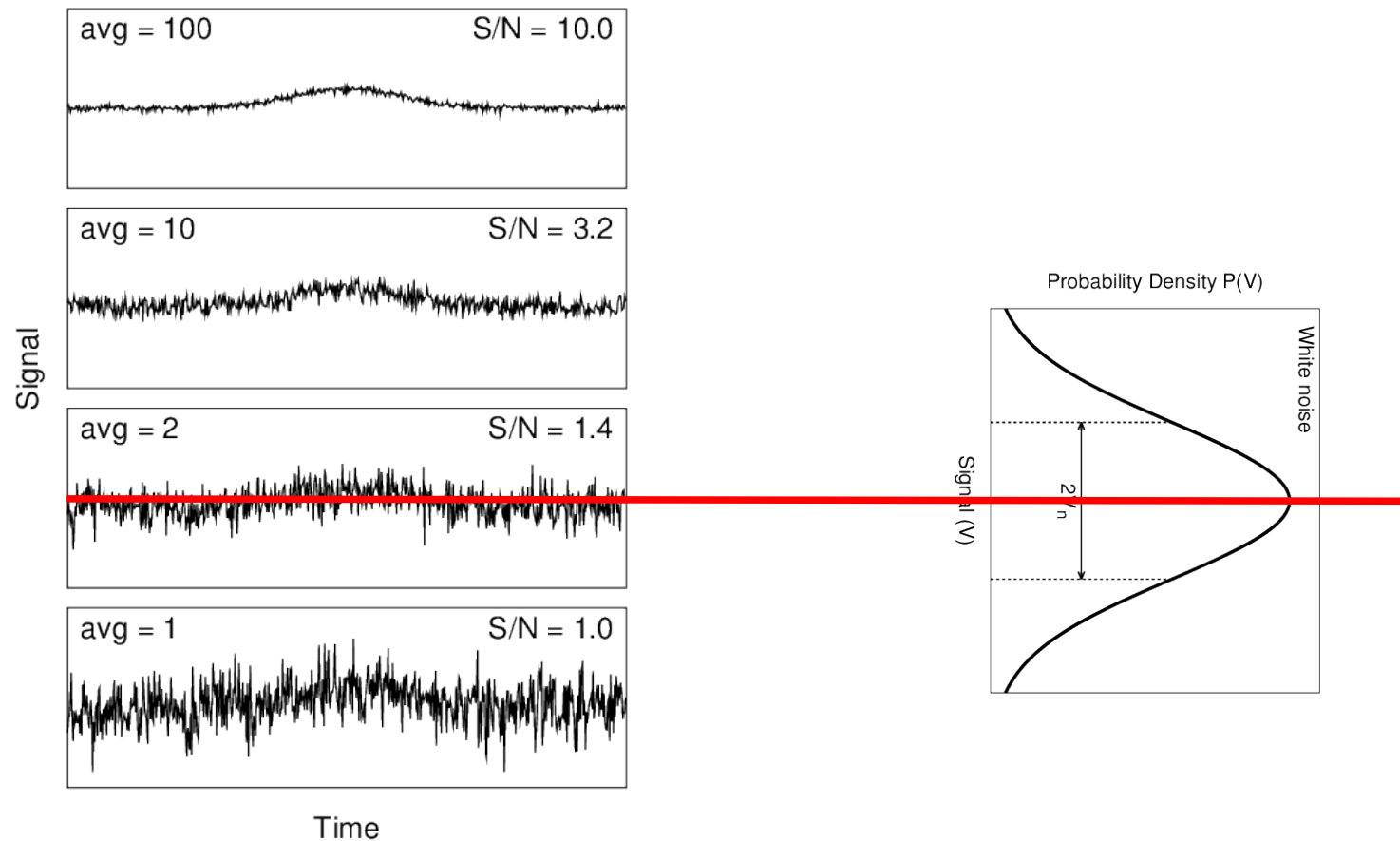


The signal-to-noise ratio, S/N or SNR is the power ratio between the two (how much power,  $\sim V^2$ , is in them).

# Signal, Noise and S/N

**Noise** is random (or looks like it to us). It has a certain **probability function**.

It also often has a certain **frequency spectrum** (which shows the correlation between measurement points)

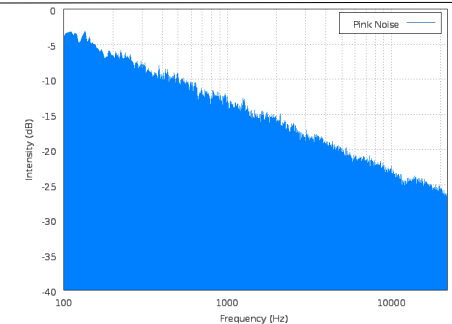


# Noise types

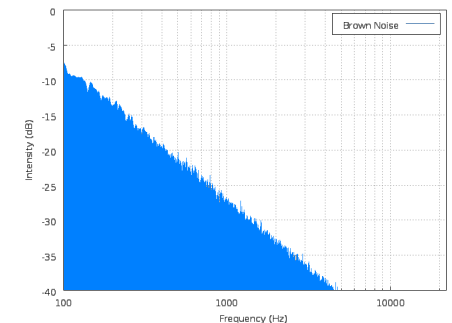
**White Noise:** Spectrum: flat ( $\sim 1$ ). Equally loud at all frequencies



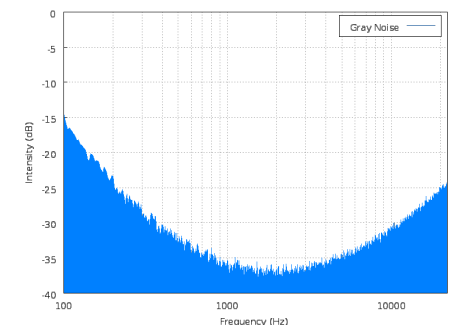
**Pink Noise:** Spectrum:  $\sim 1/f$ . Lower frequencies louder



**Brown(ian) Noise:** Caused by random motion of particles. Spectrum:  $\sim 1/f^2$



**Gray Noise:** Spectrum is inverse of sensitivity spectrum of ear (sounds equally loud at all frequencies)



Frequency →

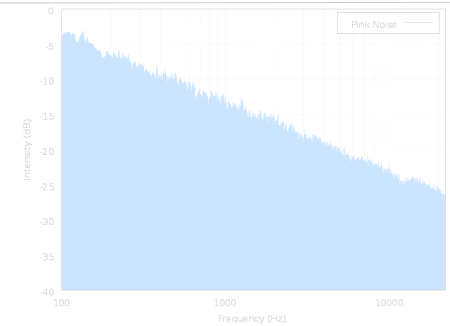


# White noise

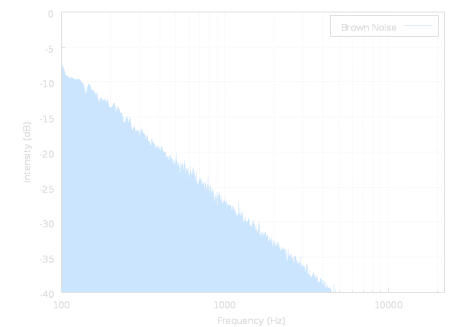
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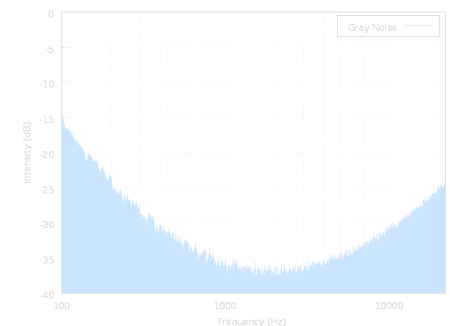
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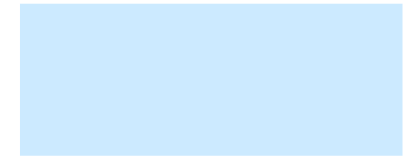
**Gray Noise:** Spectrum is inverse of sensitivity spectrum of ear (sounds equally loud at all frequencies)



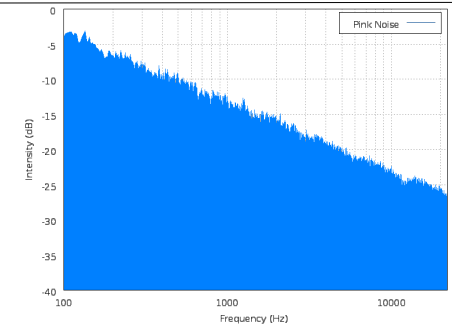
Frequency →

# Pink noise

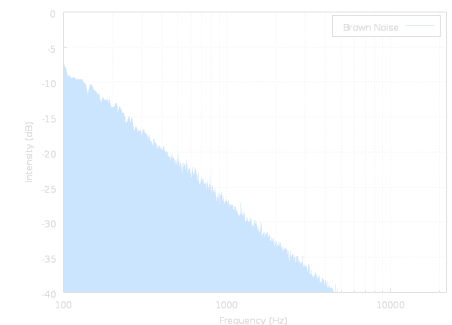
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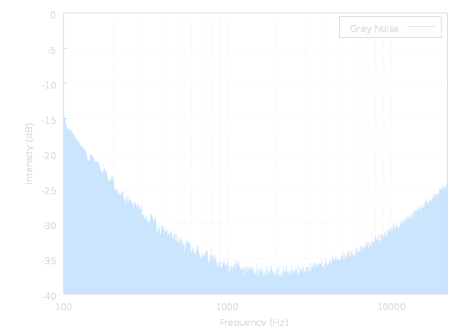
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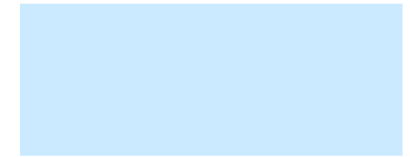
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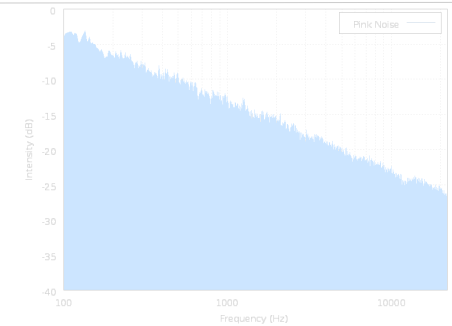
Frequency →

# Brown noise

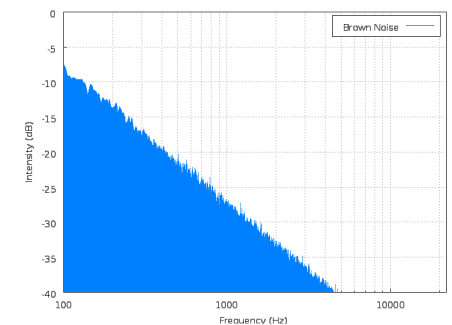
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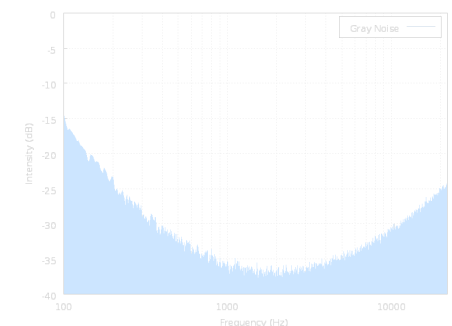
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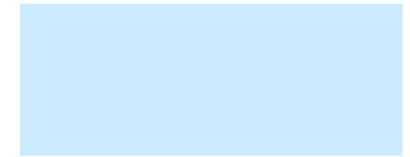
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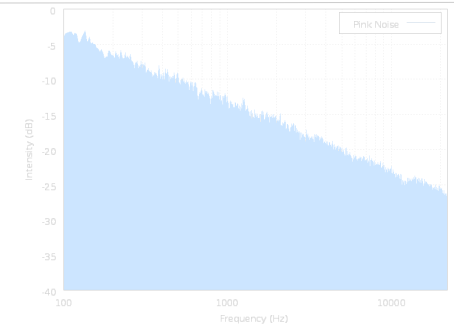
Frequency →

# Gray noise

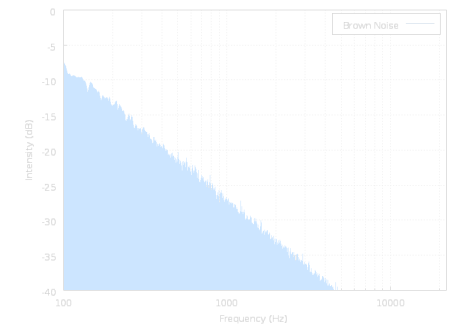
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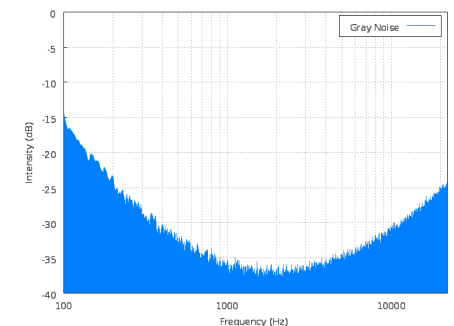
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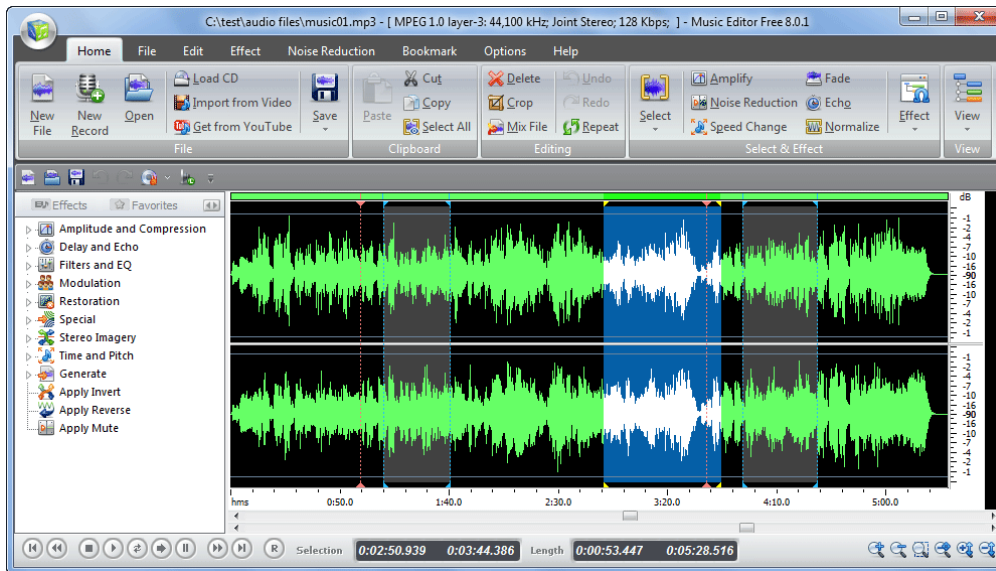


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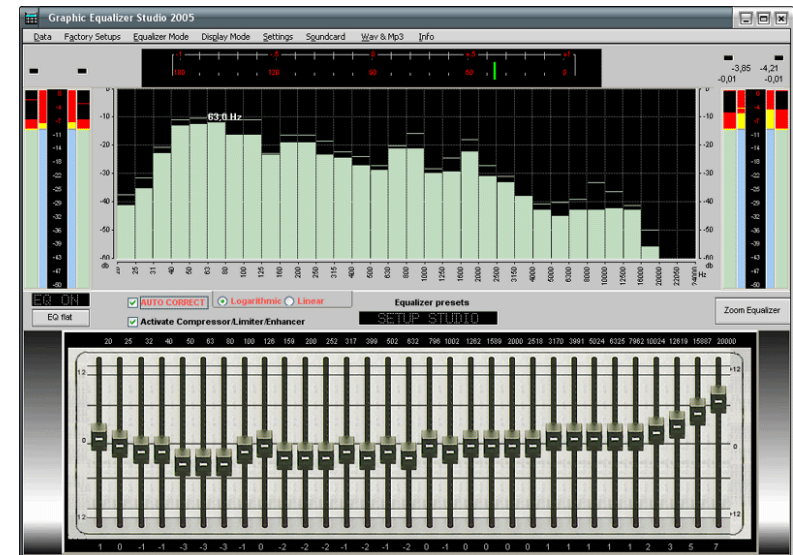


Frequency →

# Time vs. frequency



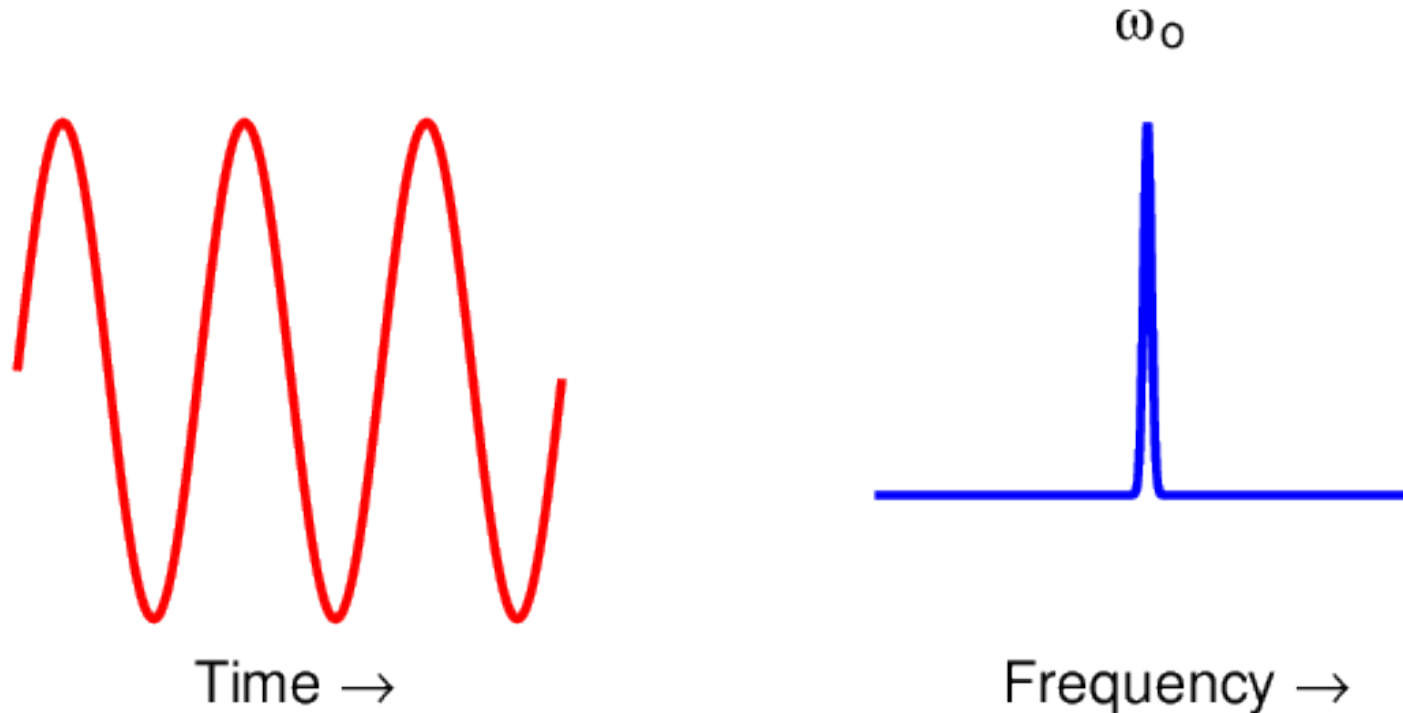
Signal in time  
Example:  $V(t) = \cos(\omega t)$



Signal in frequency  
 $V(\omega) = f(\omega)$

# Fourier/Laplace Transform

To convert from time domain to frequency domain we can use **Laplace Transform** and **Fourier Transform**



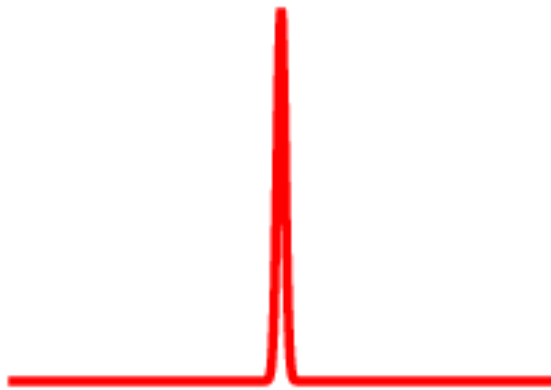
Example:  $V(t) = \cos(\omega_0 t)$

$$V(\omega) = \delta(\omega - \omega_0)$$

Single-frequency signal → Single-frequency spectrum (duh!)

# Fourier/Laplace Transform

To convert from time domain to frequency domain we can use **Laplace Transform** and **Fourier Transform**



Time →

Example:  $V(t) = \delta(t-t_0)$



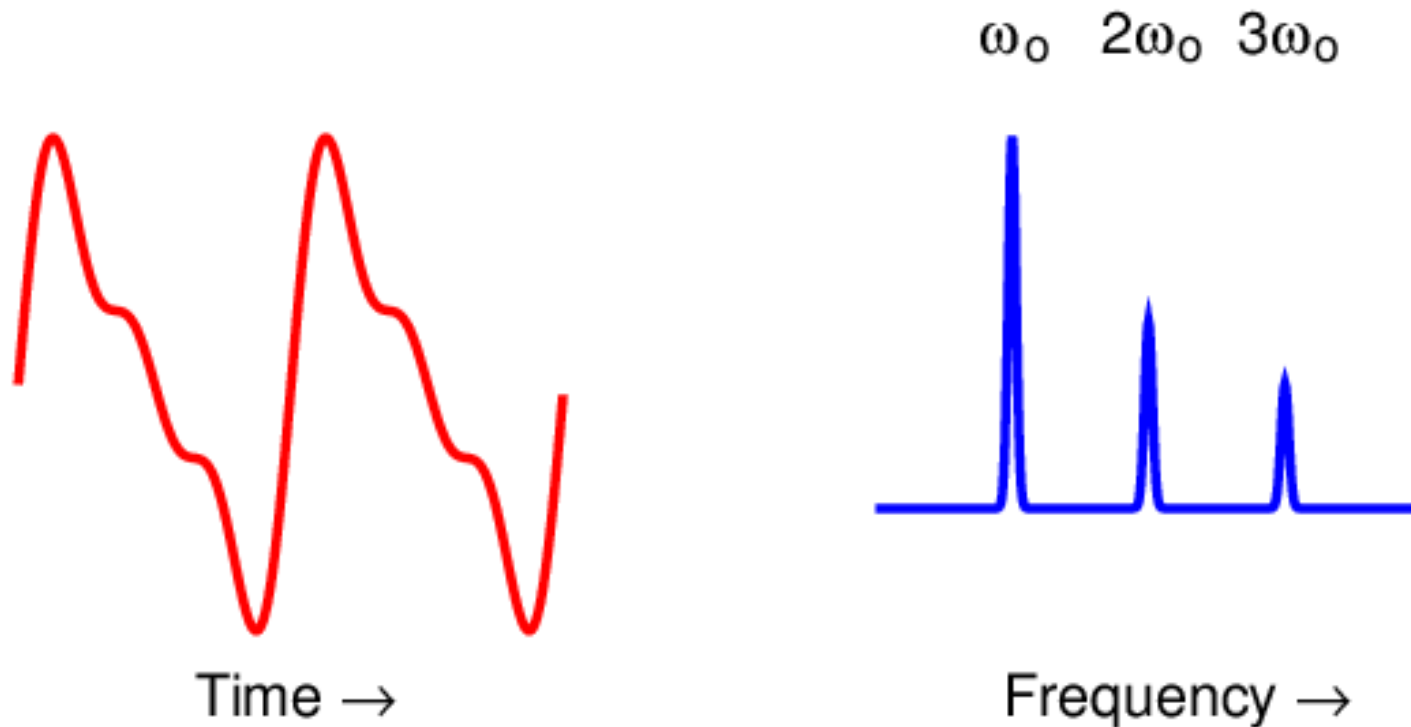
Frequency →

$V(\omega) = 1$

'Spike' signal → Constant spectrum!!

# Fourier/Laplace Transform

Fourier Transform for **periodic signals** ( $Ae^{if(\omega)\omega t}$ )

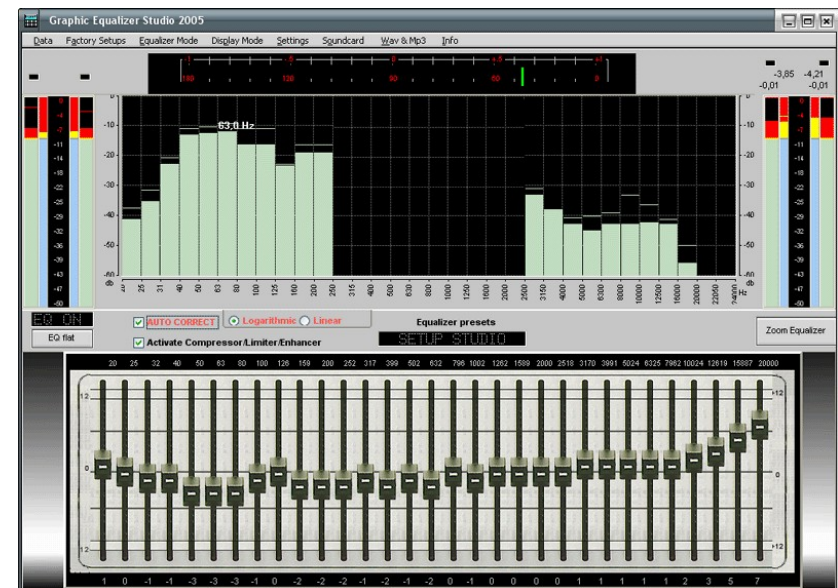
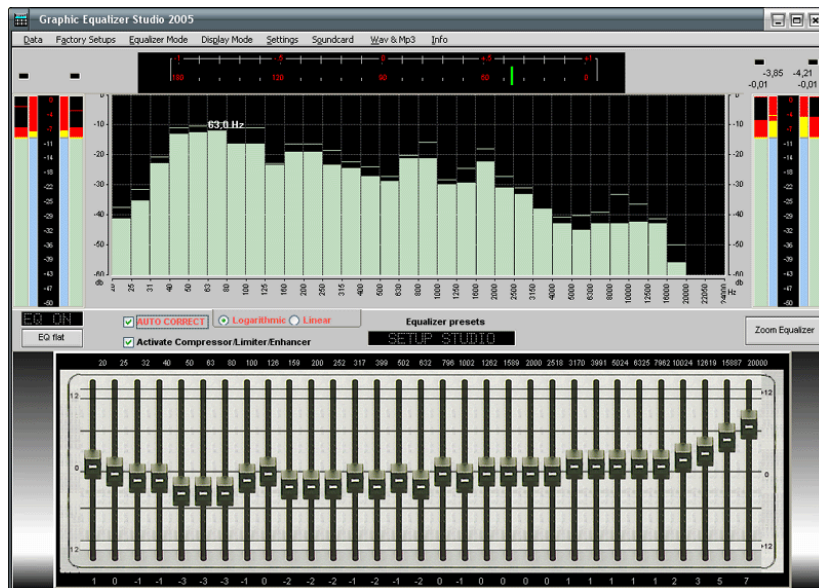


Periodic signal  $\rightarrow \omega, 2\omega, 3\omega, 4\omega, 5\omega, \dots$  etc.  
 $\omega = 2\pi/T$



# Filtering

**Filtering** then is the process of letting through only part of the spectrum

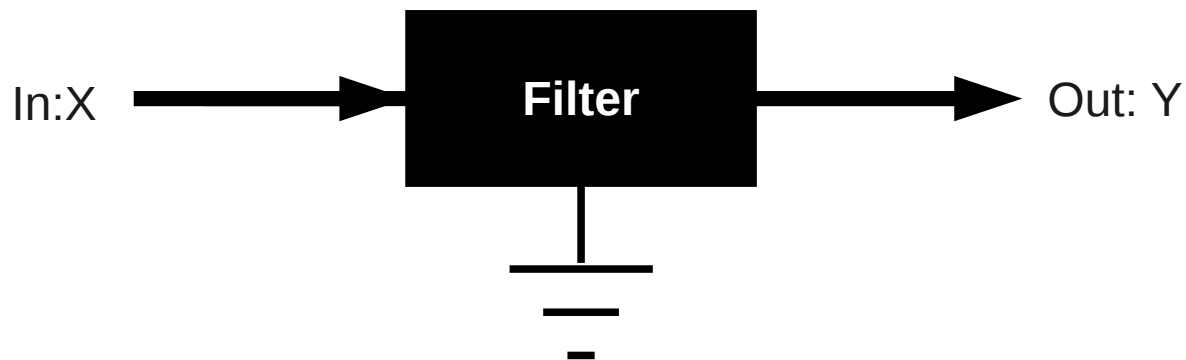
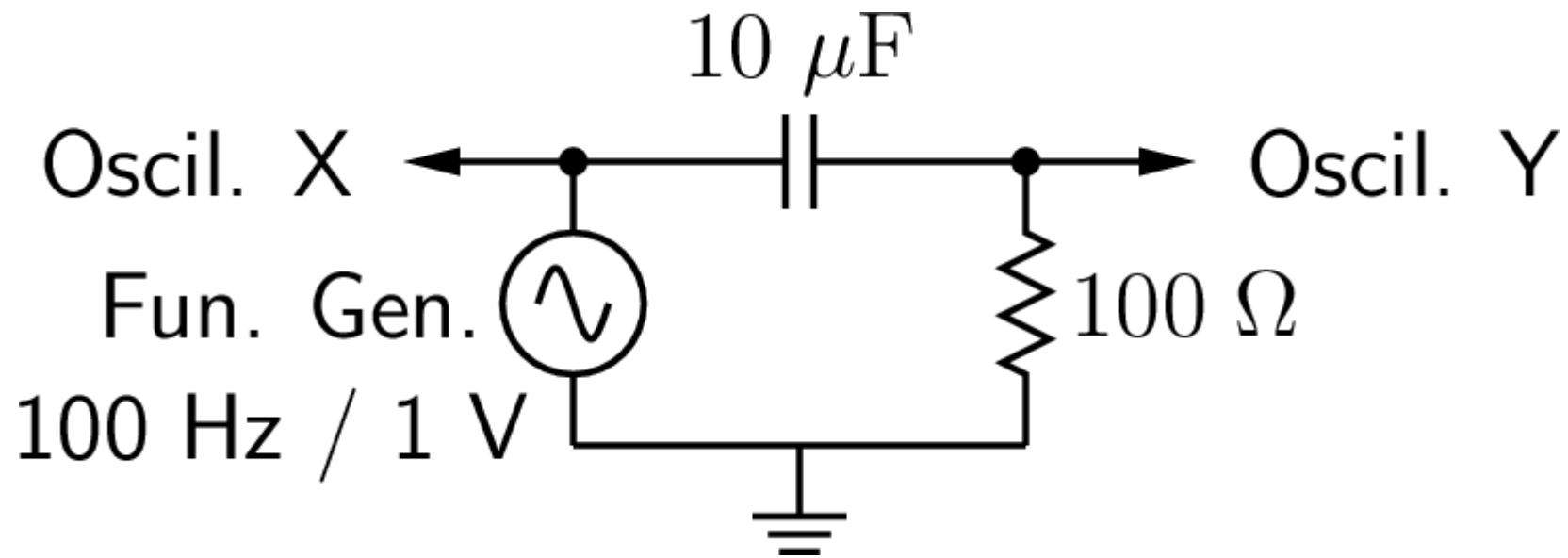


Example of a 'notch' filter (blocking part of the spectrum)

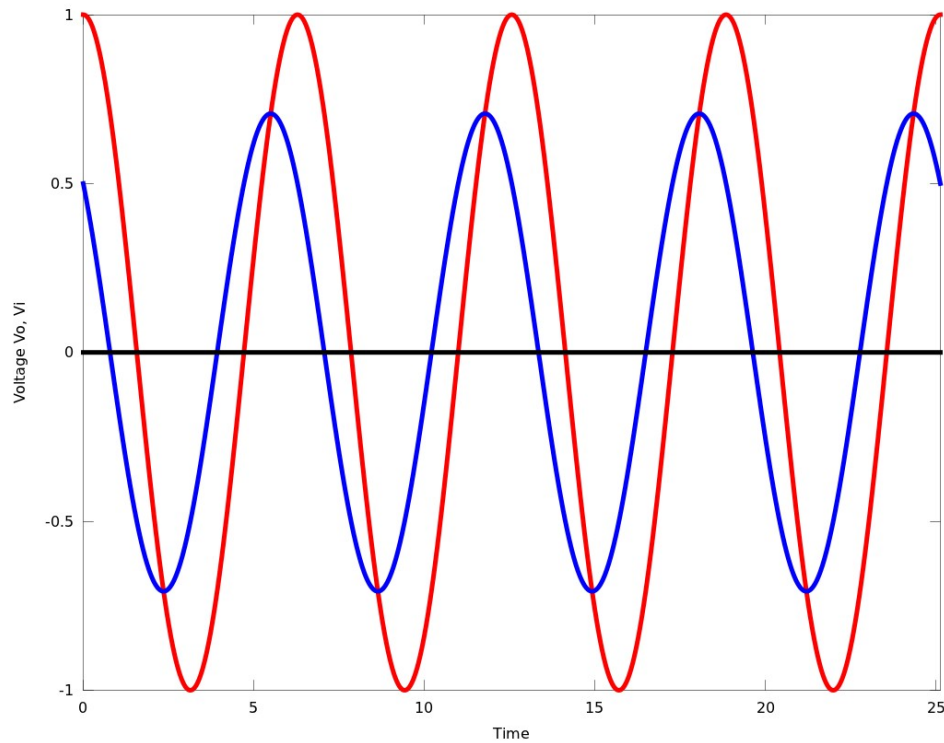
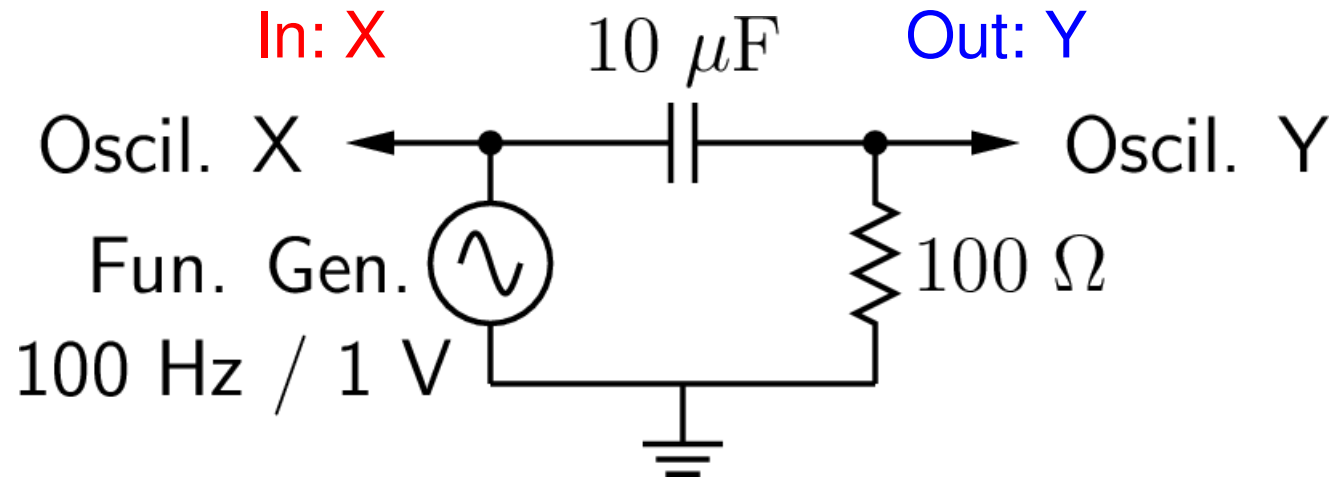
# Simple pass-filters

Simple examples of filters.

Practical lecture on Analog Electronics:



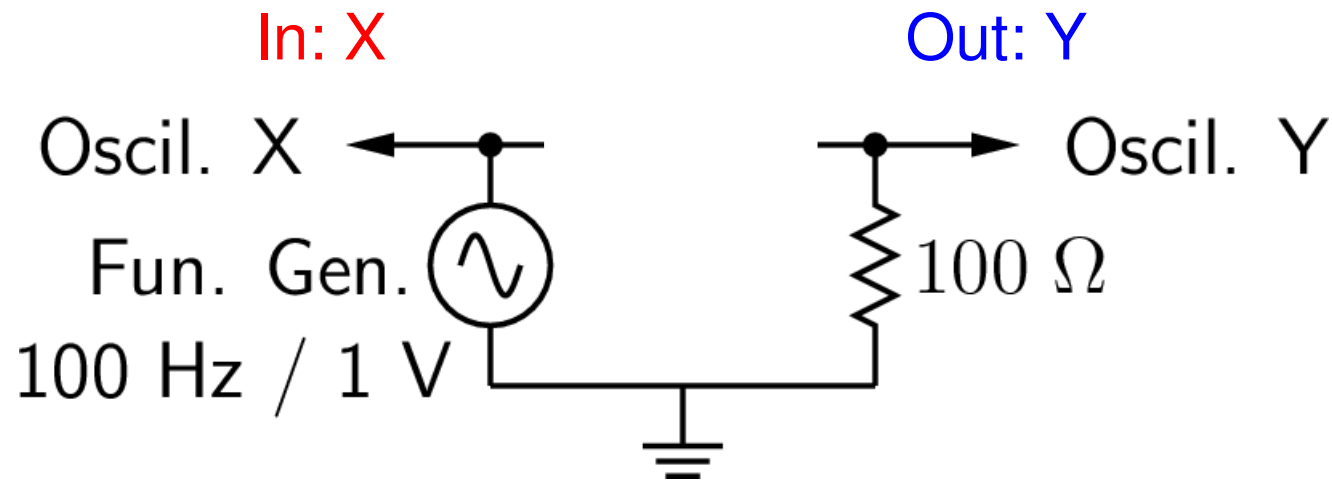
# Simple Low-Pass Filter



In: X

Out: Y

# Simple Low-Pass Filter

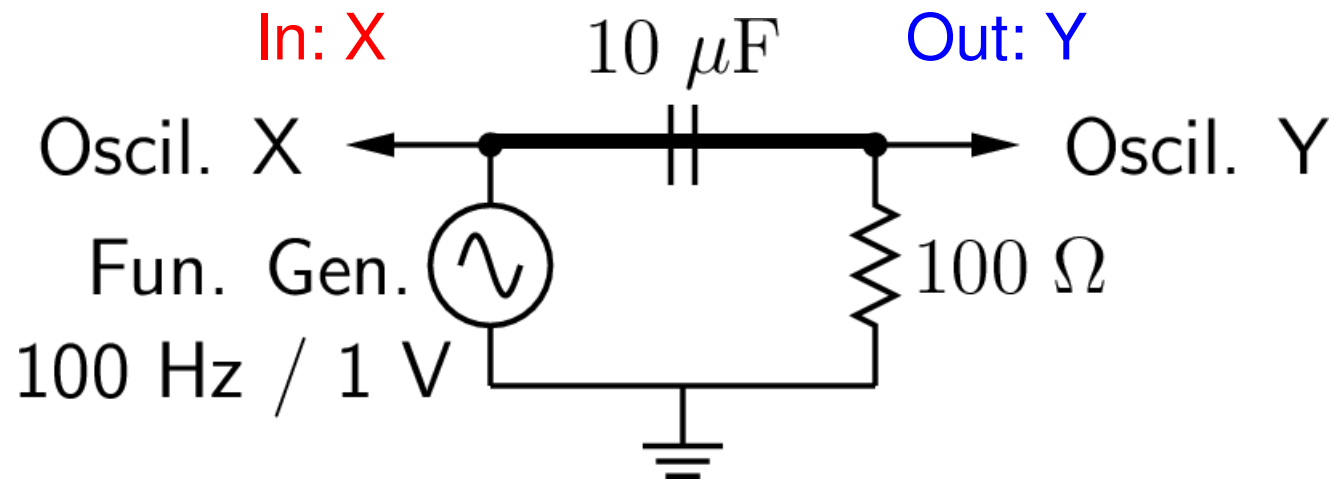


Capacitor is open circuit for low frequencies!!

$$\omega = 0. Y (\text{out}) = 0$$

$$Z_c = \frac{1}{i\omega C}$$

# Simple Low-Pass Filter

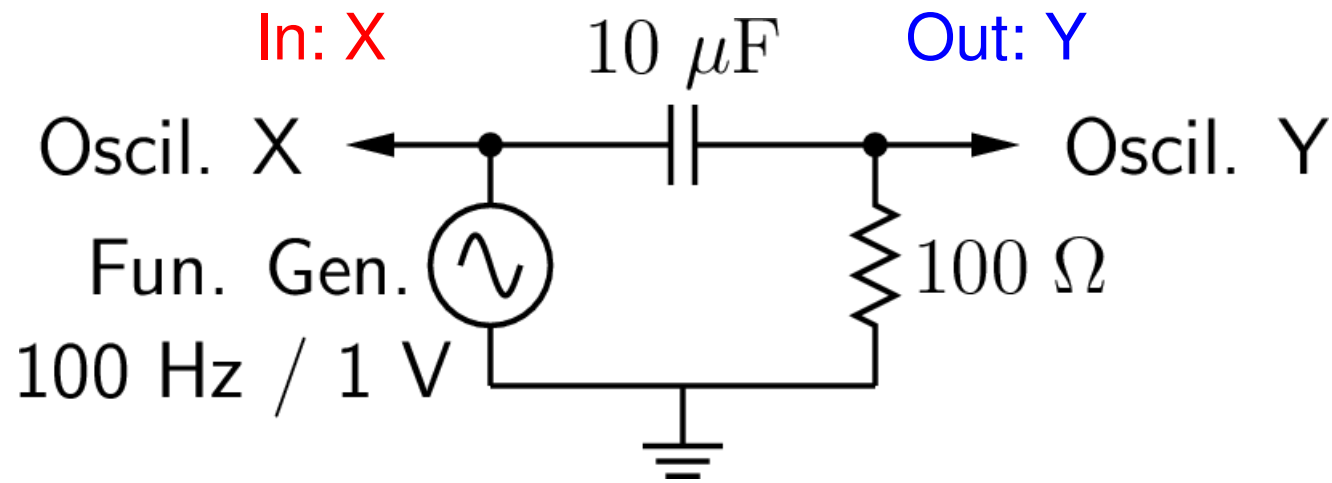


Capacitor is short circuit for high frequencies!!

$$\omega = \infty. Y (\text{out}) = X (\text{in})$$

$$Z_C = \frac{1}{i\omega C}$$

# Simple Low-Pass Filter

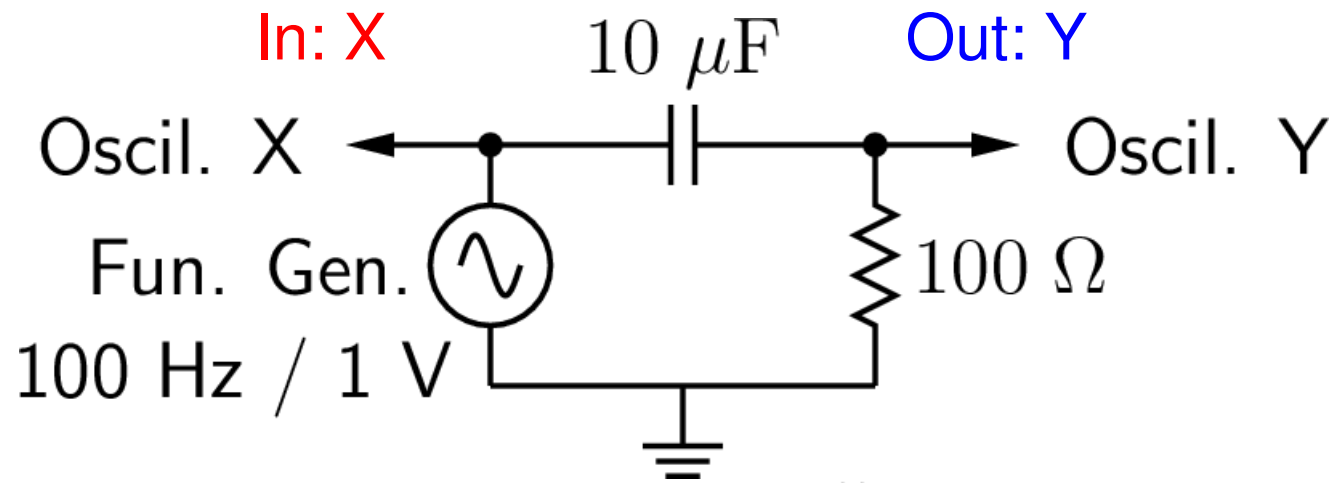


$$\frac{V_o}{V_i} = \frac{R}{R + 1/i\omega C} = \frac{1}{1 + 1/i\omega RC}$$

$$\omega = 0: V_o/V_i = 0$$

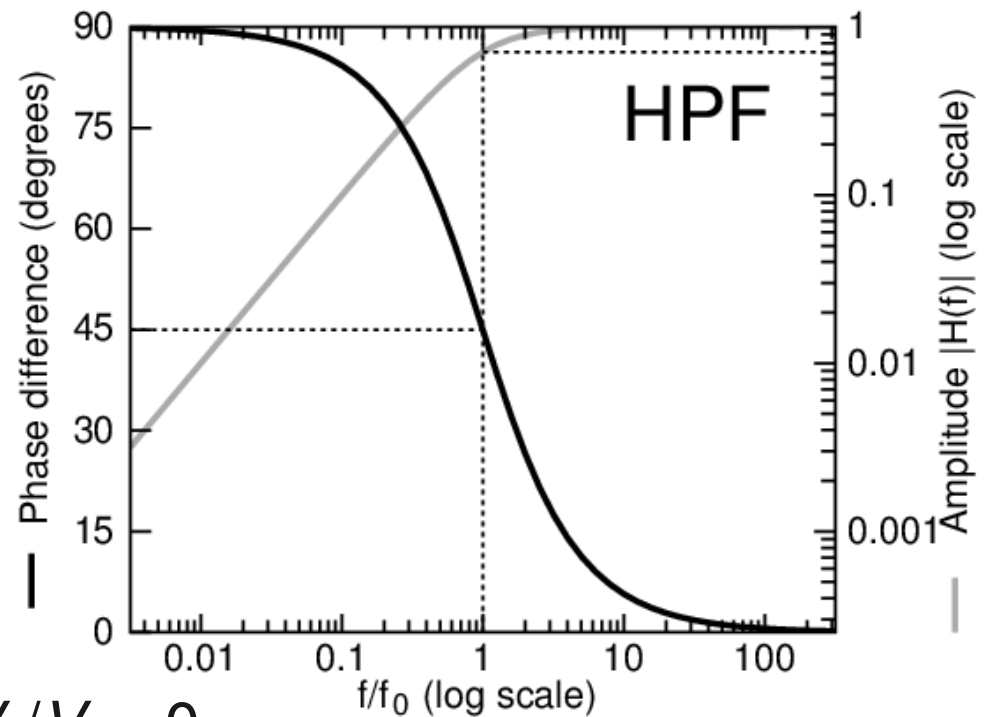
$$\omega = \infty: V_o/V_i = 1$$

# Simple Low-Pass Filter



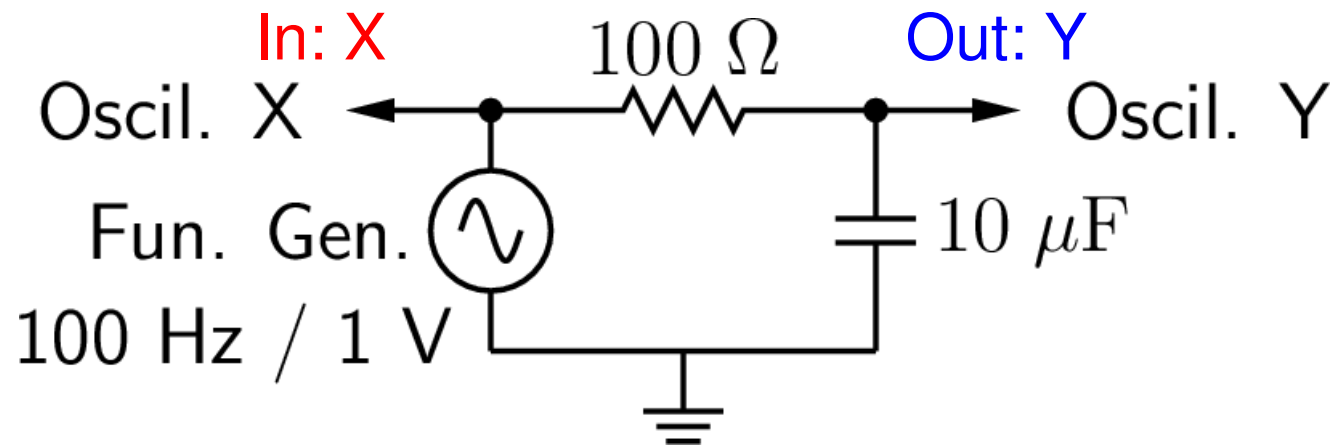
$$\omega = \infty: V_o/V_i = 1$$

$$\frac{V_o}{V_i} = \frac{1}{1 + 1/i\omega RC}$$



$$\omega = 0: V_o/V_i = 0$$

# Simple Low-Pass Filter



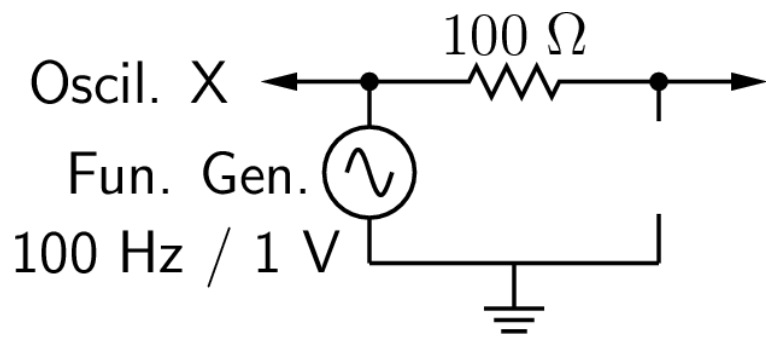
$$\frac{V_o}{V_i} = \frac{1/i\omega C}{R + 1/i\omega C} = \frac{1}{1 + i\omega RC}$$

$$\omega = 0: V_o/V_i = 1$$

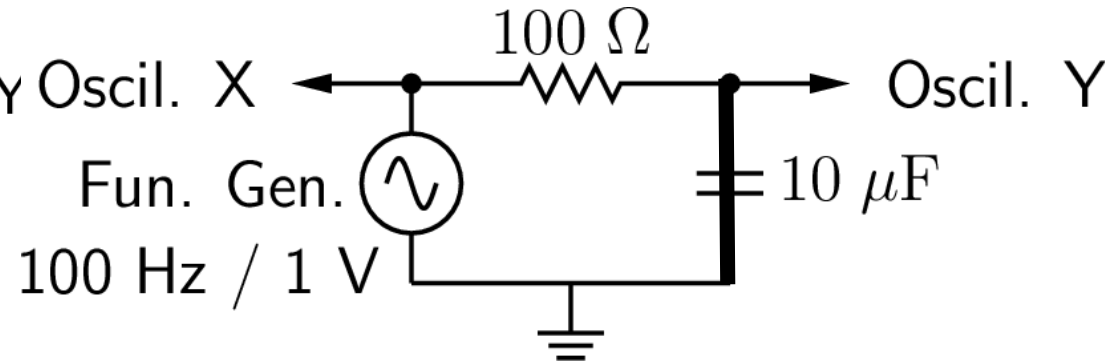
$$\omega = \infty: V_o/V_i = 0$$



# Simple Low-Pass Filter



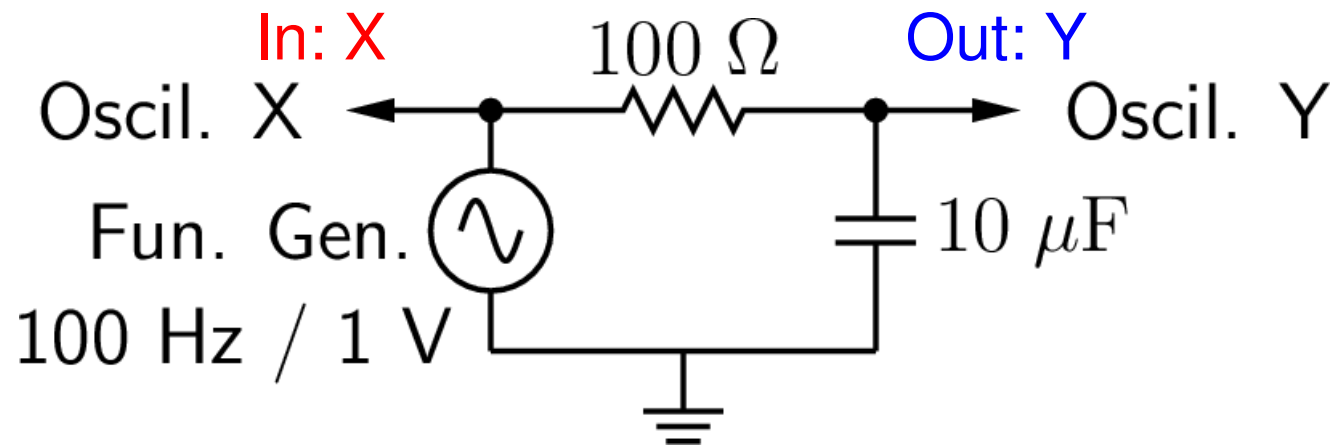
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$$\omega = \infty: V_o/V_i = 0$$

$$\frac{V_o}{V_i} = \frac{1/i\omega C}{R + 1/i\omega C} = \frac{1}{1 + i\omega RC}$$

# Simple Low-Pass Filter



$$\omega = 0: V_o/V_i = 1$$

$$\frac{V_o}{V_i} = \frac{1}{1 + i\omega RC}$$

